

Complete all problems in this packet. Work, as discussed in class, must be shown in order to receive full credit. Examples of some types of problems are included to help you.

OBJECTIVE: Graphing a parabola using the vertex and axis of symmetry

MATERIALS: Graph paper

- The graph of a quadratic function, $y = ax^2 + bx + c$, where $a \neq 0$, is a parabola.
- The axis of symmetry is the line $x = -\frac{b}{2a}$.
- The x -coordinate of the vertex is $-\frac{b}{2a}$. The y -coordinate of the vertex is $y = f\left(-\frac{b}{2a}\right)$, or the y -value when $x = -\frac{b}{2a}$.
- The y -intercept is $(0, c)$.

Example

Graph $y = 2x^2 - 8x + 5$.

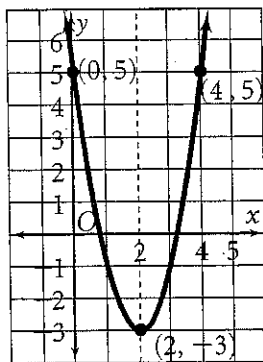
$$x = -\frac{b}{2a} = \frac{-(-8)}{2(2)} = \frac{8}{4} = 2$$

x -coordinate of vertex: 2

$$\begin{aligned} f\left(-\frac{b}{2a}\right) &= f(2) = 2(2)^2 - 8(2) + 5 \\ &= 8 - 16 + 5 \\ &= -3 \end{aligned}$$

y -coordinate of vertex: -3

y -intercept: $(0, 5)$



← Find the equation of the axis of symmetry.

$$\leftarrow -\frac{b}{2a}$$

← Find the y -value when $x = 2$.

← The vertex is at $(2, -3)$.

← The y -intercept is at $(0, c) = (0, 5)$.

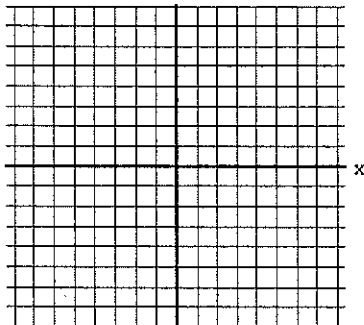
← Since a is positive, the graph opens upward, and the vertex is at the bottom of the graph. Plot the vertex and draw the axis of symmetry. Plot $(0, 5)$ and its corresponding point on the other side of the axis of symmetry.

Exercises

Graph each parabola. Label the vertex and the axis of symmetry.

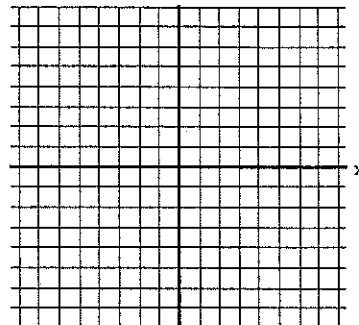
1. $y = x^2 - 4x + 7$

y



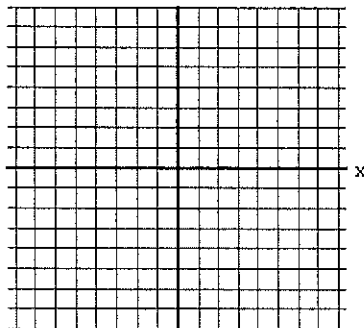
2. $y = x^2 + 8x + 11$

y



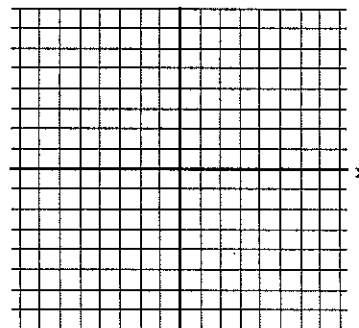
3. $y = -3x^2 + 6x - 9$

y



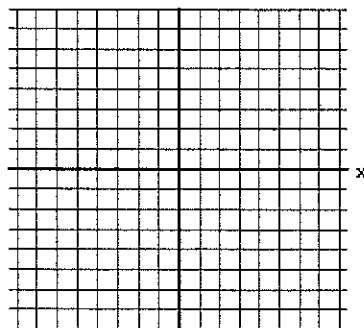
4. $y = -x^2 - 8x - 15$

y



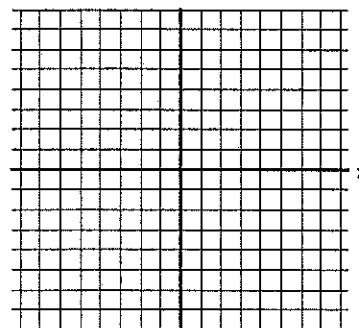
5. $y = 2x^2 - 8x + 1$

y



6. $y = -2x^2 - 12x - 7$

y



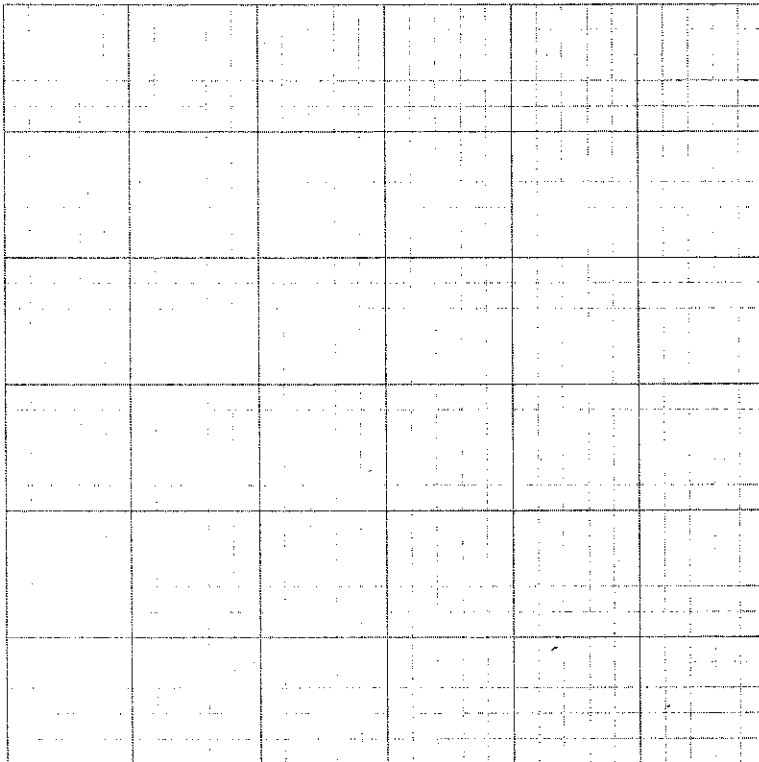
Minimum Cost In 27–29, use the following information.

A manufacturer of lighting fixtures has daily production costs modeled by

$$y = 0.25x^2 - 10x + 800$$

where y is the total cost in dollars and x is the number of fixtures produced.

27. Sketch the graph of the model. Label the vertex.



28. What is the minimum daily production cost, y ?

29. How many fixtures should be produced each day to yield a minimum cost?

Example

Factor the expression $6x^2 - 5x - 4$.

$$a = 6, b = -5, \text{ and } c = -4$$

← Find a , b , and c ; they are the coefficients of each term.

$$ac = -24 \text{ and } b = -5$$

← We are looking for factors with product ac and sum b .

Factors of -24	1, -24	-1, 24	2, -12	-2, 12	3, -8	-3, 8	4, -6	-4, 6
Sum of factors	-23	23	-10	10	-5	5	-2	2

The factors 3 and -8 are the combination whose sum is -5.

$$6x^2 + 3x - 8x - 4$$

← Rewrite the middle term using the factors you found.

$$3x(2x + 1) - 4(2x + 1)$$

← Find common factors by grouping the terms in pairs.

$$(3x - 4)(2x + 1)$$

← Rewrite using the Distributive Property.

Check: $(3x - 4)(2x + 1)$

← You can check your answer by multiplying it back together.

$$6x^2 + 3x - 8x - 4$$

$$6x^2 - 5x - 4$$

Remember that not all quadratic expressions are factorable.

Exercises

Factor each expression.

1. $x^2 + 6x + 8$

2. $x^2 - 4x + 3$

3. $2x^2 - 6x + 4$

4. $2x^2 - 11x + 5$

5. $2x^2 - 7x - 4$

6. $4x^2 + 16x + 15$

7. $x^2 - 5x - 14$

8. $7x^2 - 19x - 6$

9. $x^2 - x - 72$

OBJECTIVE: Solving quadratic equations by factoring

MATERIALS: None

When graphing a quadratic equation, remember to use the formula

$h = -\frac{b}{2a}$ to find the x -coordinate of the vertex of a parabola.

To complete the graph, plot the y -intercept $(0, c)$ and then make the parabola symmetrical.

Example

Solve the quadratic equation $x^2 + 6x + 8 = 0$ by factoring.

Factoring

Step 1

Factor the equation.

$$(x + 4)(x + 2) = 0$$

Step 2

Solve each factor for x .

$$x + 4 = 0 \text{ or } x + 2 = 0$$

$$x = -4 \text{ or } x = -2$$

Exercises

Solve each quadratic equation by factoring.

7. $x^2 - 7x + 12 = 0$

8. $2x^2 + x - 15 = 0$

9. $x^2 + x - 2 = 0$

10. $3x^2 - 5x + 2 = 0$

11. $x^2 + 5x + 6 = 0$

12. $x^2 + x - 20 = 0$

OBJECTIVE: Adding, subtracting, and multiplying complex numbers

MATERIALS: None

- A *complex number* consists of a real part and an imaginary part. It is written in the form $a + bi$, where a and b are real numbers.
- When adding or subtracting complex numbers, you combine the real parts and then combine the imaginary parts.
- When multiplying complex numbers, use the Distributive Property.
- $i^2 = (\sqrt{-1})(\sqrt{-1}) = -1$ and $i = \sqrt{-1}$

Examples

Simplify $(3 - i) + (2 + 3i)$.

$$(3 - i) + (2 + 3i)$$

$$= \textcircled{3} - \boxed{i} + \textcircled{2} + \boxed{3i}$$

← Circle real parts. Put a square around imaginary parts.

$$= (3 + 2) + (-1 + 3)i$$

← Combine.

$$= 5 + 2i$$

Simplify $(3 + 4i)(5 + 2i)$.

$$(3 + 4i)(5 + 2i)$$

$$= 3(5) + 3(2i) + 4i(5) + 4i(2i)$$

← Use the Distributive Property.

$$= 15 + 6i + 20i + 8i^2$$

← Combine real parts and imaginary parts.

$$= 15 + 26i + 8(-1)$$

← Substitute $i^2 = -1$.

$$= 7 + 26i$$

Exercises

Simplify each expression.

1. $2i + (-4 - 2i)$

2. $5i \cdot 12i$

3. $(2 + i)(2 - i)$

4. $(3 + i)(2 + i)$

5. $(4 + 3i)(1 + 2i)$

6. $3i(1 - 2i)$

7. $(6i)(-4i)$

8. $3i(4 - i)$

9. $3 - (-2 + 3i) + (-5 + i)$

In 1–8, write the number using the imaginary unit i .

1. $\sqrt{-16}$

2. $\sqrt{-36}$

3. $\sqrt{-121}$

4. $-\sqrt{-64}$

In 9–16, use the fact that $i^2 = -1$ to simplify the expression.

9. $3i^2$

10. $(3i)^2$

11. $-5i^2$

12. $(-5i)^2$

13. $(\sqrt{-4})^2$

14. $(\sqrt{-49})^2$

15. $(\sqrt{-13})^2$

16. $(\sqrt{-11})^2$

OBJECTIVE: Solving quadratic equations by using the Quadratic Formula

MATERIALS: None

Follow each step below to solve any quadratic equation by using the Quadratic Formula.

1. Write the equation in the standard form $ax^2 + bx + c = 0$.

2. Substitute a -, b -, and c -values into the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

3. Simplify. Use imaginary numbers if necessary.

4. Check the solution(s) by substituting the values into the original equation.

Example

Use the Quadratic Formula to solve $x^2 + 2 = -2x$. Check your solution.

$$x^2 + 2 = -2x$$

$$x^2 + 2x + 2 = 0$$

← Write in standard form.

$$\underline{1}x^2 + \underline{2}x + \underline{2} = 0$$

← Underline a , circle b , and put a square around c .

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(2)}}{2(1)}$$

← Substitute 1 for a , 2 for b , and 2 for c into the Quadratic Formula.

$$= \frac{-2 \pm \sqrt{-4}}{2}$$

← Simplify to find the values of x .

$$= \frac{-2 \pm 2i}{2}$$

$$= -1 \pm i$$

Exercises

Solve each equation using the Quadratic Formula.

1. $x^2 - 3x + 2 = 0$

2. $-x^2 + 5x = 9$

3. $10x - 6 = 5x^2$

4. $x + 2x^2 + 1 = -1 - x$

5. $2x^2 + x = 10$

6. $2x + 1 = 2x^2$

In 7–12, find the discriminant, $b^2 - 4ac$, of the quadratic equation.

7. $x^2 - x + 3 = 0$

8. $-x^2 + 2x - 1 = 0$

9. $3x^2 + x - 2 = 0$

In 13–18, find the discriminant and use it to determine the number of real solutions of the equation.

13. $x^2 - 2x - 3 = 0$

14. $x^2 + 5x + 2 = 0$

15. $-x^2 + 3x - 5 = 0$

16. $-4x^2 + 20x - 25 = 0$

17. $3x^2 - 2x + 1 = 0$

18. $2x^2 - x + 4 = 0$

Throwing an Object on the Moon In 33–35, use the following information.

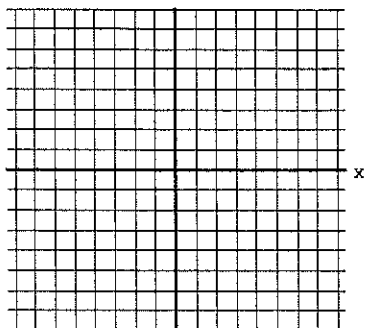
An astronaut standing on the surface of the moon throws a rock into the air with an initial velocity of 27 feet per second. The astronaut's hand is 6 feet above the surface of the moon. The height of the rock is given by $h = -2.7t^2 + 27t + 6$.

33. How many seconds is the rock in the air?

In 8–16, sketch the graph of the inequality.

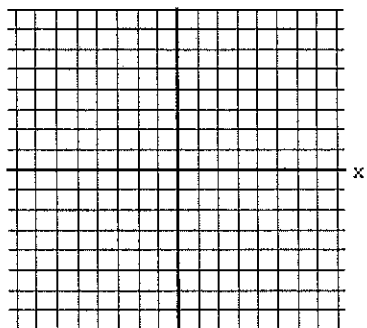
8. $y \leq 2x^2 + 1$

y



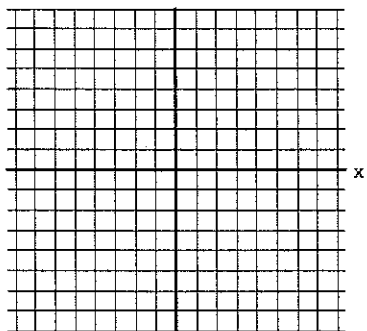
9. $y \geq x^2 + 2x$

y



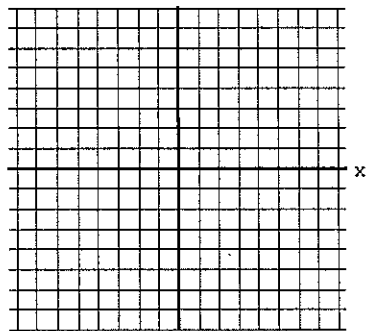
10. $y > 3x^2 - 6x$

y



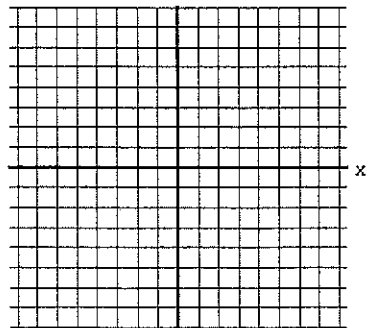
11. $y < x^2 - 2x + 1$

y



12. $y \leq -x^2 + 6x - 7$

y



13. $y \geq 3x^2 + 6x + 2$

y

